(b) Use convolution theorem to find the inverse Laplace transform of

$$\frac{1}{s^3(s^2+1)}.$$

(c) Apply Laplace transform technique to solve

$$\frac{dx}{dt}+y=\sin t,\ \frac{dy}{dt}+x=\cos t;\ x(0)=2,\ y(0)=0.$$

6. (a) Find the half range cosine series expansion of $f(x) = x - x^2$ in $0 < x < 1$.

(b) Solve :

$$(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z.$$

(c) Solve :

$$\frac{\partial^2 z}{\partial x^2}-3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=e^{x+y}+\cos(x+2y).$$

7. (a) Apply method of separation of variables to solve $\frac{\partial^2 z}{\partial x\partial y}=e^{-x}\cos y$, given that $z = 0$ when $x = 0$ and $\frac{\partial z}{\partial x} = 0$ when $y = 0$.

(b) Find the temperature distribution in a rod of length 2 m whose end points are fixed at temperature zero and the initial temperature distribution is $f(x) = 100\,x$.

(c) Solve :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,$$

subject to boundary conditions $u(0, y) = 0 = u(\pi, y)$, and $u(x, 0) = u_0$, $\lim_{y\to\infty} u(x, y) = 0$, $0 < x < \pi$.

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610 **Roll No.** | | | | | | | | | | |

B. Tech.

(SEM. II) THEORY EXAMINATION 2011-12

MATHEMATICS—II

Time : 3 Hours *Total Marks : 100*

Note :— Attempt ***all*** questions.

SECTION—A

1. Attempt **all** parts of this question :— **(10×2=20)**

(a) Find the general solution of $(2D + 1)^2y = 0$, where $D=\frac{d}{dt}$.

(b) Find the particular integral of $\frac{d^2y}{dx^2}-y=x^2$.

(c) Evaluate $\int_{-1}^{1} x^2 P_2(x)\,dx$.

(d) Evaluate $\int_0^{\pi/2} \sqrt{\pi x}\, J_{1/2}(2x)\,dx$.

(e) Find the Laplace transform of $f(t) = t^4 e^{2t}$.

(f) Find the function whose Laplace transform is $\frac{e^{-\pi s}}{s^2+2}$.

(g) Find the constant term if the function $f(x) = x + x^2$ is expanded in Fourier series defined in $(-1, 1)$.

(h) Find the particular integral of

$(D^2 + DD')\,z = \sin(x + y)$ where $D=\frac{\partial}{\partial x}$, $D'=\frac{\partial}{\partial y}$.

(i) Find the steady state temperature distribution in a rod of length 20 cm whose ends are kept at 40°C and 60°C respectively.

(j) Find the general solution of $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ using method of separation of variables.

SECTION—B

2. Attempt any **three** parts of the following :— (3×10=30)

(a) Apply method of variation of parameter to solve

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x.$$

(b) Use Frobenius series method to find the series solution of

$$(1 - x^2)y'' - xy' + 4y = 0.$$

(c) Solve $y'' + 3y' + 2y = te^{-t}$, $y(0) = 1$, $y'(0) = 0$, using Laplace transform method.

(d) Find the Fourier series expansion of the function

$$f(x) = \begin{cases} -1, & \text{for } -\pi < x < -\pi/2 \\ 0, & \text{for } -\pi/2 < x < \pi/2 \\ 1, & \text{for } \pi/2 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots.$$

(e) Find the deflection of the vibrating string of unit length whose end points are fixed if the initial velocity is zero and the initial deflection is given by

$$u(x,0) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ -1, & \frac{1}{2} < x \le 1 \end{cases}$$

SECTION—C

Note :— Attempt any **two** parts from each question of this section. [(2×5)×5=50]

3. (a) Solve the following differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2 e^{-x} \cos x.$$

(b) Solve the following equation by reducing into normal form

$$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + (x^2 - 8)y = x^2 e^{-x^2/2}.$$

(c) An RL circuit has an e.m.f. given (in volts) by 4 sin t, a resistance of 100 ohms, an inductance of 4 henries and no initial current. Find the current at any time t.

4. (a) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials.

(b) Solve the following differential equation in terms of Bessel's function

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(8 - \frac{1}{x^2}\right)y = 0.$$

(c) Prove that :

(i) $P_n(1) = 1$,

(ii) $\frac{d}{dx}\left\{x^n J_n(x)\right\} = x^n J_{n-1}(x)$.

5. (a) Find the Laplace transform of $f(t) = \frac{e^{-t} \sin t}{t}$.